

- Elementary Geometry.

Geometric Constructions

We discuss construction via knowledge of locus of points. examples, a circle is a locus of points which are equidistant from a given point; locus of points which are equidistant from the ends of a given segment is the perpendicular bisector of that segment; the locus of points lying inside an angle and equidistant from its sides is the bisector of that angle. The essence of the method is that a locus of points is constructed that satisfy each of the conditions and the sought after point belongs to the intersection of these loci. Simplest example here is constructing a triangle with three given sides. One draws a segment and them two circles with radius as the other two sides and the intersection of these two circles is the third point of the triangle. More standard well known loci are, loci of points equidistant from a given line and specified distance are two parallel lines. Locus of points from which the segment is visible at a right angle is a circle with punctured ends.

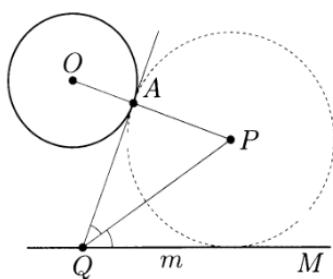


Figure 1:

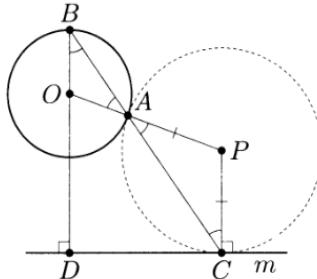


Figure 2:

Illustrative Example 1 Construct a right angled triangle given the hypotenuse (c) and the length of the altitude (h) to the hypotenuse. **Solution** Refer to the figure 1. We draw the hypotenuse, then a circle with the hypotenuse as diameter and a line parallel at distance h and the intersection of these loci is the third point (C) of the triangle.

Illustrative Example 2 Construct a circle tangent to a given line m and touching externally a given circle at a given point A on the circle. **Solution** Refer to the figure 1 and figure 2 for two solutions. **Solution 1:** Extend line OA . Draw a tangent to the circle at A that intersects the line m at Q , bisect the angle AQM and this help in locating the centre of the required circle at P . **Solution 2:** Draw a line perpendicular to the line m and passing through O . This cuts the circle at B . Join BA which cuts the line m at C . A perpendicular at C will help locate the centre of the required circle.

Tasks **Task 1)** Explain how to build the following loci: a) removed from the given line by a given distance; b) from which the given segment is visible at a given angle (consider three cases: the given angle is right, acute, obtuse) **Task 2)** Explain how to construct a tangent to a circle passing through a given point (consider two cases, point on the circle and point outside the circle). **Task 3)** Explain how to build a triangle given the side, the median to it and the radius of the circumscribed circle to the triangle. **Task 4)** Explain how to construct a circle that touches a given line m at a given point B and passes through a given point A not lying on the line m . **Task 5)** Three points A , B and C are given. Explain how to construct three circles externally tangent to each

other at these points. **Task 6)** A circle and a straight line m are given, which does not intersect it. Explain how to construct a circle that is tangent to this circle and the given straight line at a given point Q on the line m . **Task 7)** Explain how to construct a straight line passing through a given point M inside a given angle such that a triangle with a given perimeter is constructed.