11.2 Simplifying Expressions

In this section we look at how to simplify expressions, in particular, how to remove brackets from both formulae and equations.

Collecting like terms

Examples

$$a + a + a = 3a$$

$$a + b + a = 2a + b$$

$$2y + 8y = 10y$$

$$x + x^2 + x^2 = x + 2x^2$$

Only like terms can be collected

Example 1

Simplify the following expressions,

(a) $4a + 2b + 3a + 6b$ (b) $3x - 4y + 2x - y$

(c) $x^2 + 4x + 2x^2 - x$ (d) $4a^2 + a + 2a^2 - 3a$

Solution

(a) $4a + 2b + 3a + 6b = 7a + 8b$ (b) $3x - 4y + 2x - y = 5x - 5y$

(c) $x^2 + 4x + 2x^2 - x = 3x^2 + 3x$ (d) $4a^2 + a + a^2 - 3a = 6a^2 - 2a$

Expanding Brackets

Every term in each bracket must be multiplied by every other item.

$$x(4x + 2) = x \times 4x + x \times 2$$

$$= 4x^2 + 2x$$

$$(x + 1)(x + 4) = x \times x + x \times 4 + 1 \times x + 1 \times 4$$

$$= x^2 + 4x + x + 4$$

$$= x^2 + 5x + 4$$

Alternatively, you can expand brackets using the 'box' method, as shown opposite.

$(x + 1)(x + 4) = x^2 + 1x + 4x + 4 = x^2 + 5x + 4$

$\times$	x	$+1$
x	x^2	$+1x$
$+4$	$+4x$	$+4$

Example 2

Expand each of the following:

(a) $2(x+3)$ (b) $4(2x-6)$

(c) $x(x+2)$ (d) $2x(3x-2)$

Solution

(a) $2(x+3) = 2\times x + 2\times 3$

$= 2x + 6$

(b) $4(2x-6) = 4\times 2x - 4\times 6$

$= 8x - 24$

(c) $x(x+2) = x\times x + x\times 2$

$= x^2 + 2x$

(d) $2x(3x-2) = 2x\times 3x - 2x\times 2$

$= 6x^2 - 4x$

Example 3

Expand,

(a) $(x+6)(x+3)$ (b) $(x+4)(2x-5)$

Solution

(a) $(x+6)(x+3) = x\times x + x\times 3 + 6\times x + 6\times 3$

$= x^2 + 3x + 6x + 18$

$= x^2 + 9x + 18$

or alternatively, using the box method,

$\times$	x	$+6$
x	x^2	$+6x$
$+3$	$+3x$	$+18$

$(x+6)(x+3) = x^2 + 6x + 3x + 18 = x^2 + 9x + 18$

(b) $(x+4)(2x-5) = x\times 2x - x\times 5 + 4\times 2x - 4\times 5$

$= 2x^2 - 5x + 8x - 20$

$= 2x^2 + 3x - 20$

Again, using the box method,

×	x	$+4$
$2x$	$2x^2$	$+8x$
-5	$-5x$	-20

$$(x+4)(2x-5) = 2x^2 + 8x - 5x - 20 = 2x^2 + 3x - 20$$

Exercises

1. Simplify each of the following by collecting like terms:

   (a) $4a + b + 2a$ (b) $4b + 2c + 6b + 3c$

   (c) $4a + 5b - a + 2b$ (d) $14p + 11q - 8p + 3q$

   (e) $6x - 4y + 8x + 9y$ (f) $11x + 8y + 3z - 2y + 4z$

   (g) $16x - 8y - 3x - 4y$ (h) $11y + 12z - 10y + 4z + 2y$

2. Simplify each of the following:

   (a) $3x + 3x^2 + 4x - x^2$ (b) $4y^2 + 4y - 2y^2 + 3y$

   (c) $a^2 + a + 3a^2 - 2a$ (d) $6x^2 + 12x - 9x^2 + 3x$

3. Expand each of the following expressions by multiplying out the brackets:

   (a) $3(x+6)$ (b) $4(x+2)$ (c) $3(x-1)$

   (d) $4(2x+5)$ (e) $6(3x-5)$ (f) $7(2x-5)$

   (g) $6(4-2x)$ (h) $8(3-5x)$ (i) $9(5x+10)$

4. Simplify each of the following expressions:

   (a) $2(x+3)+4(x+4)$ (b) $5(x-6)+2(x+3)$

   (c) $4(6-x)+7(2x+1)$ (d) $11(x-2)+4(7x+3)$

   (e) $8(x-6)+4(7-x)$ (f) $3(4-5x)+6(3x-2)$

5. Expand each of the following expressions by multiplying out the brackets:

   (a) $x(x+3)$ (b) $x(6x+1)$ (c) $x(3x-2)$

   (d) $2x(4-x)$ (e) $6x(2x+4)$ (f) $5x(3x-7)$

   (g) $11x(x-3)$ (h) $14x(2+3x)$ (i) $6x(4-2x)$

6. Expand each of the following expressions by multiplying out the brackets:

(a) $(x+4)(x+3)$ (b) $(x+2)(x+4)$ (c) $(x+1)(x+5)$

(d) $(x+6)(x-1)$ (e) $(x-4)(x+2)$ (f) $(x-3)(x+2)$

(g) $(x-4)(x-5)$ (h) $(x-3)(x-2)$ (i) $(x-7)(x-9)$

7. Simplify each of the following expressions:

(a) $(x+2)(x+4)+(x+1)(x+2)$

(b) $(x+3)(x+7)+(x-1)(x+5)$

(c) $(x+6)(x+2)-(x-2)(x+3)$

(d) $(x-4)(x-8)-(x-1)(x-9)$

8. Expand each expression:

(a) $(2x+1)(3x+2)$ (b) $(4x-7)(2x+1)$

(c) $(3x+5)(2x-8)$ (d) $(4x+5)(3x-8)$

(e) $(8x+2)(3x-3)$ (f) $(6x-5)(3x-7)$

9. Simplify:

(a) $(3x+2)(5x+9)+(4x-2)(3x-5)$

(b) $(4x+6)(5x+1)-(2x+3)(3x+1)$

(c) $(6x-5)(x+1)-(2x+7)(3x-5)$

10. Expand:

(a) $(x+1)^2$ (b) $(x-2)^2$ (c) $(x+3)^2$

(d) $(x+5)^2$ (e) $(x-7)^2$ (f) $(x-8)^2$

(g) $(x+10)^2$ (h) $(x-12)^2$ (i) $(x+4)^2$

(j) $(2x+3)^2$ (k) $(4x-7)$ (l) $(3x+2)^2$

(m) $(4x+1)^2$ (n) $(5x-2)^2$ (o) $(6x-4)^2$

11. Expand:

(a) $(x+1)(x-1)$ (b) $(x+3)(x-3)$

(c) $(x+7)(x-7)$ (d) $(x+9)(x-9)$

(e) $(x+12)(x-12)$ (f) $(2x+1)(2x-1)$

(g) $(3x+2)(3x-2)$ (h) $(4x+7)(4x-7)$

12. Expand:

(a) $(x+1)^3$ (b) $(2x+1)^3$ (c) $(x-5)^3$

13. Here are some algebra cards:

$n+2$	$n \div 2$	n^2	$2+n$
	n		$2n$
$n-2$		$n+n$	
	n^3		$2n-n$

(a) One of the cards will always give the same answer as $\frac{n}{2}$.

Which card is it?

(b) One of the cards will always give the same answer as $n \times n$.

Which card is it?

(c) *Two* of the cards will always give the same answer as $2 \times n$.

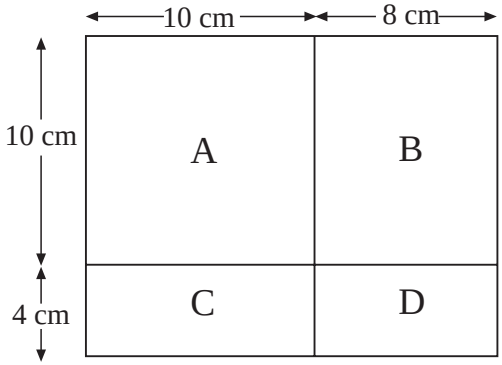
Which cards are they?

(d) Write a *new* card which will always give the same answer as

$3n+2n$.

(KS3/97/Ma/Tier 5-7/P1)

14. (a) (i) The diagram shows a rectangle 18 cm long and 14 cm wide.
It has been split into *four smaller rectangles,* A, B, C and D.
Write down the *area* of each of the small rectangles.
One has been done for you.

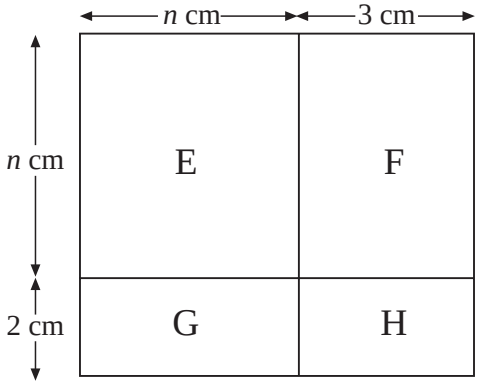


Area of Rectangle C $= 40$ cm^2.

(ii) What is the area of the *whole* rectangle?

(iii) What is 18×14 ?

(b) (i) The diagram shows a rectangle $(n+3)$ cm long and $(n+2)$ cm wide.
It has been split into *four smaller rectangles.*
Write down a *number* or an *expression* for the *area* of *each small rectangle.*
One has been done for you.



Area of Rectangle F $= 3n$ cm^2.

(ii) What is $(n+3)(n+2)$ multiplied out?

(KS3/99/Ma/Tier 5-7/P1)

15. Multiply out and simplify these expressions:

(a) $3(x-2)-2(4-3x)$

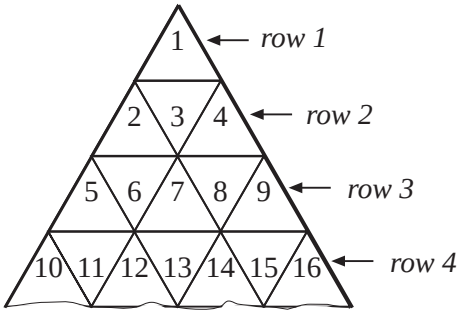
(b) $(x+2)(x+3)$

(c) $(x+4)(x-1)$

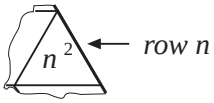
(d) $(x-2)^2$

(KS3/98/Ma/Tier 6-8/P1)

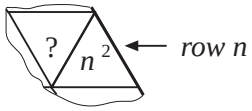
16. A number grid is inside a large triangle.
The small triangles are numbered consecutively.
The diagram shows the first 4 rows.



(a) An expression for the *last* number in row n is n^2.



Write an expression for the *last but one* number in row n.



(b) An expression for the *first* number in row n is n^2-2n+2.
Calculate the value of the first number in *row 10*.

(c) Make a copy of the table and complete it by writing an expression:

first number in row n	n^2-2n+2
second number in row n	

(d) Make a copy of the table and complete it by writing an expression:

centre number in *row* n	n^2-n+1
centre number in *row*	$(n+1)^2-(n+1)+1$

(e) Multiply out and simplify the expression $(n+1)^2-(n+1)+1$.
Show your working.

(KS3/99/Ma/Tier 6-8/P1)

11.3 Factorising

In this section we consider examples of the process of factorising, whereby the process of removing brackets is reversed and brackets are introduced into expressions.

Example 1

Factorise:

(a) $8x + 12$ (b) $35x + 28$

Solution

(a) Note that both terms are multiples of 4, so we can write,

$$8x + 12 = 4(2x + 3)$$

(b) Here both terms are multiples of 7, so

$$35x + 28 = 7(5x + 4)$$

Results like this can be checked by multiplying out the bracket to get back to the original expression.

Example 2

Factorise,

(a) $x^2 + 2x$ (b) $3x^2 - 9x$ (c) $x^3 - x^2$

Solution

(a) Here, as both terms are multiples of x, we can write,

$$x^2 + 2x = x(x + 2)$$

(b) In this case, both terms are multiples of x and 3, giving,

$$3x^2 - 9x = 3x(x - 3)$$

(c) In this example, both terms are multiples of x^2,

$$x^3 - x^2 = x^2(x - 1)$$

Sometimes it is possible to factorise in stages. For example, in part (b), you could have worked like this:

$$\begin{aligned} 3x^2 - 9x &= 3(x^2 - 3x) \\ &= 3x(x - 3) \end{aligned}$$

Example 3

Factorise:

(a) $x^2 + 9x + 18$ (b) $x^2 + 2x - 15$ (c) $x^2 - 7x + 12$

Solution

(a) This expression will need to be factorised into two brackets:

$$x^2 + 9x + 18 = (x \quad)(x \quad)$$

As the expression begins x^2, both brackets must begin with x. The two numbers to go in the brackets must multiply together to give 18 and add to give 9. So they must be 3 and 6, giving,

$$x^2 + 9x + 18 = (x + 3)(x + 6)$$

You can check this result by multiplying out the brackets.

(b) We note first that two brackets are needed and that both must contain an x, as shown:

$$x^2 + 2x - 15 = (x \quad)(x \quad)$$

Two other numbers are needed which, when multiplied give – 15 and when added give 2. In this case, these are – 3 and 5. So the factorisation is,

$$x^2 + 2x - 15 = (x - 3)(x + 5)$$

Check this result by multiplying out the brackets.

(c) Again, we begin by noting that,

$$x^2 - 7x + 12 = (x \quad)(x \quad)$$

We require two numbers which, when multiplied give 12 and when added give – 7. In this case, these numbers are – 3 and – 4.

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

Exercises

1. Factorise:

(a)	$4x - 2$	(b)	$6x - 12$	(c)	$5x - 20$
(d)	$4x + 32$	(e)	$6x - 8$	(f)	$8 - 12x$
(g)	$21x - 14$	(h)	$15x + 20$	(i)	$30 - 10x$

2. Factorise:

(a)	$x^2 + 4x$	(b)	$x^2 - 3x$	(c)	$4x - x^2$
(d)	$6x^2 + 8x$	(e)	$9x^2 + 15x$	(f)	$7x^2 - 21x$
(g)	$28x - 35x^2$	(h)	$6x^2 - 14x$	(i)	$5x^2 - 3x$

3. Factorise:

(a) $x^3 + x^2$ (b) $2x^2 - x^3$ (c) $4x^3 - 2x^2$

(d) $8x^3 + 4x^2$ (e) $16x^2 - 36x^3$ (f) $4x^3 + 22x^2$

(g) $16x^2 - 6x^3$ (h) $14x^3 + 21x^2$ (i) $28x^3 - 49x^2$

4. (a) Expand $(x + 5)(x - 5)$.

(b) Factorise $x^2 - 25$.

(c) Factorise each of the following:

(i) $x^2 - 49$ (ii) $x^2 - 64$ (iii) $x^2 - 100$

(iv) $x^2 - a^2$ (v) $x^2 - 4b^2$

5. Factorise:

(a) $x^2 + 7x + 12$ (b) $x^2 + 8x + 7$ (c) $x^2 + 11x + 18$

(d) $x^2 + 12x + 27$ (e) $x^2 + 17x + 70$ (f) $x^2 + 6x + 8$

(g) $x^2 + 16x + 28$ (h) $x^2 + 18x + 77$ (i) $x^2 + 16x + 63$

6. Factorise:

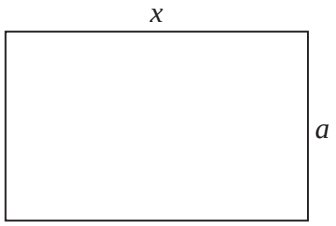
(a) $x^2 + x - 2$ (b) $x^2 + x - 20$ (c) $x^2 - x - 12$

(d) $x^2 - 13x + 36$ (e) $x^2 - 10x + 16$ (f) $x^2 + x - 42$

(g) $x^2 + 13x - 30$ (h) $x^2 - 17x + 72$ (i) $x^2 - 2x - 99$

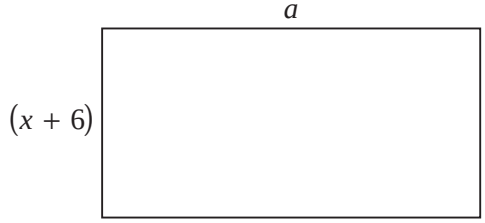
7. The area of the rectangle shown is $x^2 - 5x$.

Express a in terms of x.



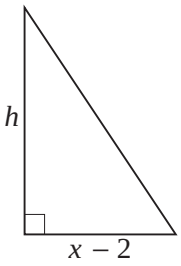
8. The area of the rectangle shown is $x^2 + 11x + 30$.

Express a in terms of x.



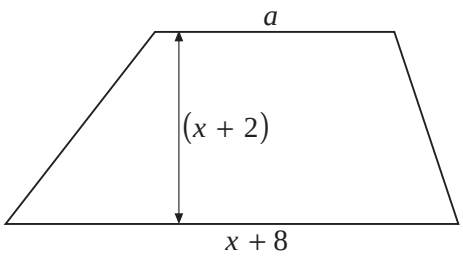
9. The area of the triangle shown is $\frac{1}{2}x^2 + \frac{3}{2}x - 5$.

Express h in terms of x.



10. The area of the trapezium shown is $\frac{1}{2}x^2 + 10x + 18$.

Determine a.



11.4 Using Formulae

In this section we make use of formulae and develop simple formulae ourselves. First we begin with some revision of working with *negative numbers*.

Example 1

If $a = 6,\ b = -5,\ c = -7$ and $d = 3,$ calculate:

(a) $a + c$ (b) $a - b$ (c) bc (d) $b^2 + cd$

Solution

(a) $a + c = 6 + (-7)$
$= 6 - 7$
$= -1$

(b) $a - b = 6 - (-5)$
$= 6 + 5$
$= 11$

(c) $bc = (-5) \times (-7)$
$= 35$

(d) $b^2 + cd = (-5)^2 + (-7) \times 3$
$= 25 + (-21)$
$= 25 - 21$
$= 4$