10.8 Simultaneous Linear Equations

A pair of equations which use both terms at the same time, such as

$$x + 2y = 8$$
$$2x + y = 7,$$

are known as a pair of *simultaneous* equations.

Worked Example 1

Solve the pair of simultaneous equations

$$x + 2y = 8$$
$$2x + y = 7.$$

Solution

First it is helpful to label the equations (1) and (2).

$$x + 2y = 8 \qquad (1)$$
$$2x + y = 7 \qquad (2)$$

Equation (1) is multiplied by 2, so that it contains the same number of x's as equation (2).

Let the new equation be labelled (3).

$$2x + 4y = 16 \qquad (3) \qquad [2 \times (1)]$$
$$2x + y = 7 \qquad (2)$$

Equation (2) is now subtracted from equation (3).

$$2x + 4y = 16 \qquad (3)$$
$$2x + y = 7 \qquad (2)$$
$$3y = 9 \qquad (3) - (2)$$

Solving $3y = 9$ gives $y = 3$.

This value of y can now be substituted into equation (1) to give:

$$x + 2 \times 3 = 8$$
$$x + 6 = 8$$

Solving this gives $x = 2$. So the solution to the equation is $x = 2$, $y = 3$.

Worked Example 2

Solve the simultaneous equations

$$3x + 5y = 2$$
$$-4x + 7y = -30.$$

Solution

First label the equations (1) and (2) as shown below.

$$3x + 5y = 2 \qquad (1)$$
$$-4x + 7y = -30 \qquad (2)$$

Then multiply equation (1) by 4 and equation (2) by 3 to make the number of x's in both equations the same.

$$12x + 20y = 8 \qquad (3) \qquad [4 \times (1)]$$
$$-12x + 21y = -90 \qquad (4) \qquad [3 \times (2)]$$

Now add together equations (3) and (4) to give

$$12x + 20y = 8 \qquad (3)$$
$$-12x + 21y = -90 \qquad (4)$$
$$41y = -82 \qquad (3) + (4)$$

Solving the equation $41y = -82$ gives $y = -2$.

This value for y can be substituted into equation (1) to give

$$3x + 5 \times (-2) = 2$$

or

$$3x - 10 = 2.$$

Solving this equation gives:

$$3x - 10 = 2$$
$$3x = 12$$
$$x = \frac{12}{3}$$
$$= 4.$$

So the solution is $x = 4$ and $y = -2$.

Note

It is a good idea to check that solutions are correct by substituting these values back into the original equations. Here,

$$3 \times 4 + 5 \times (-2) = 2$$

and

$$-4 \times 4 + 7 \times (-2) = -30$$

You must check *both* equations to make sure that you have the correct answer.

Worked Example 3

Denise sells 300 tickets for a concert. Some tickets are sold to adults at £5 each and some are sold to children at £4 each. If she collects in £1444 in ticket sales, how many tickets have been sold to adults and how many to children?

Solution

Let x = number of adults' tickets
and y = number of children's tickets.

She has sold 300 tickets, so

$$x + y = 300.$$

The value of the adult tickets sold is £5x, and the value of the children's tickets is £4y.

As the value of all the tickets sold is £1444, then

$$5x + 4y = 1444.$$

The two simultaneous equations

$$x + y = 300 \qquad (1)$$
$$5x + 4y = 1444 \qquad (2)$$

can now be solved. First multiply equation (1) by 5 and subtract equation (2) to give

$$5x + 5y = 1500 \qquad (3) \qquad [5 \times (1)]$$
$$5x + 4y = 1444 \qquad (2)$$
$$y = 56 \qquad (3) - (2)$$

This value can then be substituted into equation (1) to give

$$x + 56 = 300$$
or
$$x = 244.$$

So the solution is $x = 244$ and $y = 56$. That is, 244 adults' tickets and 56 children's tickets have been sold.

Investigation

Consider the following simultaneous equations.

$$2x + y = 6 \qquad (1)$$
$$x = 1 - \tfrac{1}{2}y \qquad (2)$$

If (2) is substituted for x into (1), then

$$2\left(1 - \tfrac{1}{2}y\right) + y = 6$$
$$2 - y + y = 6$$
$$2 = 6!$$

Find out where the problem lies.

Exercises

1. Solve each pair of simultaneous equations.

(a) $x + 2y = 5$, $3x + y = 5$

(b) $3x + 2y = 19$, $x + 5y = 15$

(c) $x - 2y = 4$, $4x + 3y = 49$

(d) $2x + 3y = 14$, $5x + 2y = 24$

(e) $3x + 4y = 2$, $7x - 5y = 9$

(f) $4x + 2y = 16$, $-3x + 2y = -19$

(g) $5x + y = 2$, $-4x + 3y = 44$

(h) $6x - 4y = 12$, $-9x + 2y = -66$

(i) $7x - 2y = 23$, $3x + 4y = 39$

(j) $8x + 4y = 7$, $-12x + 8y = -6$

(k) $4x - 2y = -0.1$, $5x + 2y = 1.5$

(l) $6x - 5y = 41$, $4x + 15y = 31$

(m) $-2x + 5y = 14$, $10x + 7y = 26$

(n) $8x + 5y = -29$, $3x - 7y = -2$

(o) $6x - 5y = -14$, $18x - 4y = 6$

(p) $6x - 8y = -2$, $5x + 2y = 1.8$

(q) $\frac{1}{2}x - \frac{1}{4}y = 0$, $\frac{1}{3}x + \frac{2}{3}y = 10$

(r) $\frac{1}{5}x - \frac{1}{10}y = -1$, $\frac{1}{4}x + \frac{1}{2}y = 10$

2. Find the coordinates of the point of intersection of the lines:

(a) $x + y = 8$ and $y = 2x - 1$

(b) $x + y = 10$ and $y = 2x + 1$

(c) $x + y = 4$ and $y = 2 - \frac{x}{10}$

3. Describe the problems you encounter when you try to solve the simultaneous equations:

$$\begin{aligned} 3x - 2y &= 8 \\ 9x - 6y &= 2. \end{aligned}$$

4. (a) Check that $x = 2$ and $y = 5$ is a solution of both the equations below.

$$\begin{aligned} x + 2y &= 12 \\ 3x + 6y &= 36 \end{aligned}$$

(b) Try to solve the equations. What happens?

(c) Write both equations in the form $y = \ldots$ and comment on the equations you obtain.